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TRANSMISSION AND REFLECTION OF PRESSURE WAVES BY COMPRESSOR AND TURBINE STAGES, BASED ON AN ACTUATOR-DISK MODEL

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The amplitudes of the pressure waves transmitted and reflected by an actuator disk due to the impingement of an incident pressure wave are calculated. Analytic expressions for the wave amplitudes are derived for the limit where the incident pressure rise is small and these formulas, as well as direct calculations, are used to estimate the effect of multiple wave reflections from a pair of actuator disks. These predictions help to explain a number of phenomena that have been observed in measurements made when shock-tube generated overpressure waves were sent into an operating engine.

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PREFACE

The work described in this report was conducted at the Calspan Corporation in Buffalo, New York. The work was performed by Professor W.J. Rae of the State University of New York at Buffalo (Consultant to Calspan) and by Mr. P.F. Batcho and Dr. M.G. Dunn of the Calspan Corporation. At the present time, Mr. Batcho is involved in a PhD program at Princeton University. The work was supported by the Defense Nuclear Agency under Contract No. DNA 001-83-C-0182. The work was performed during the period 1 June 1986 to 1 September 1987. The authors would like to acknowledge the helpful discussions with the technical monitor of this program, Lt. Col. Ronald M. Adams/SPWE.

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INTRODUCTION

Sharp pressure waves can originate in aircraft gas-turbine engines from a variety of sources, such as the changing inlet conditions due to a sudden maneuver, the ingestion of ordnance exhaust, or the detonation of a nearby explosive. These traveling waves interact with the various components inside the engine (compressor and turbine blade rows, burner assemblies, nozzles and ducts) and the result of the interaction is to produce transmitted and reflected waves. Under certain circumstances, the combined effects of the multiple waves produced by such interaction can cause major transients in the engine operation, such as rotating stall, surge, or extreme excursions in the combustor temperature.

Recent measurements (1-3) provide a detailed, time-resolved data base for several engines under carefully controlled conditions. These measurements show that a pressure wave entering the engine inlet makes its way through the various stages of the machine at approximately the speed of sound relative to the local flow speed, provided the static pressure rise is on the order of ten percent or less of the initial static pressure. However, the amplifications and attenuations of the incident pressure pulse that occur upon passage through the various components have not been explained in a satisfactory manner. The only theoretical tools available for interpreting the measurements are computer codes which attempt to calculate the one-dimensional time-dependent flow through the various stages (4-6). None of the available codes have been applied successfully to the data of References (1-3), and in fact most of them have stability and waveform restrictions which have thus far prevented their application to obtain predictions that would be helpful in understanding the measurements.

The work reported here has made some progress toward the prediction and understanding of the changes in waveform and amplitude that occur, by resorting to an analytic solution for the fundamental problem, namely, each compressor or turbine stage is represented as an actuator disk, and the solution is found for the waves that are transmitted and reflected when a given pressure wave is incident on the disk.

The report is organized as follows: Section 2 contains the actuator-disk analysis and numerical results for a number of typical cases. Section 3 presents the analytical formulas that apply for very weak waves, while Section 4 treats the problem

of multiple reflections between a pair of disks as a means of estimating how an actual engine would respond to a disturbance. Section 4 also contains a comparison with some experimental results, which are more easy to interpret in the light of the present analysis.

The real-engine experiments include a number of features that are not accounted for in this analysis, and the Section 5 presents a discussion of these features and the further research that is now made possible by the results of this work.

SECTION 2
ACTUATOR-DISK ANALYSIS

The data presented in (1-3) include detailed pressure measurements at a variety of points inside operating engines through which shock waves have been sent. Pressure histories were recorded on gages placed in positions upstream and downstream of various engine components: the inlet fan, the core compressor, etc. From these data it is possible to infer how an incident shock wave interacts with each component; the pressure-time records at the various positions reveal a very complex system of compression and expansion waves. Attempts to interpret these complex results have been thwarted in the past because of the absence of a solution for the elemental problem that arises in such a flow, namely the interaction of the shock with a single component. This fundamental problem is the one whose solution is reported herein.

2.1 ANALYSIS.

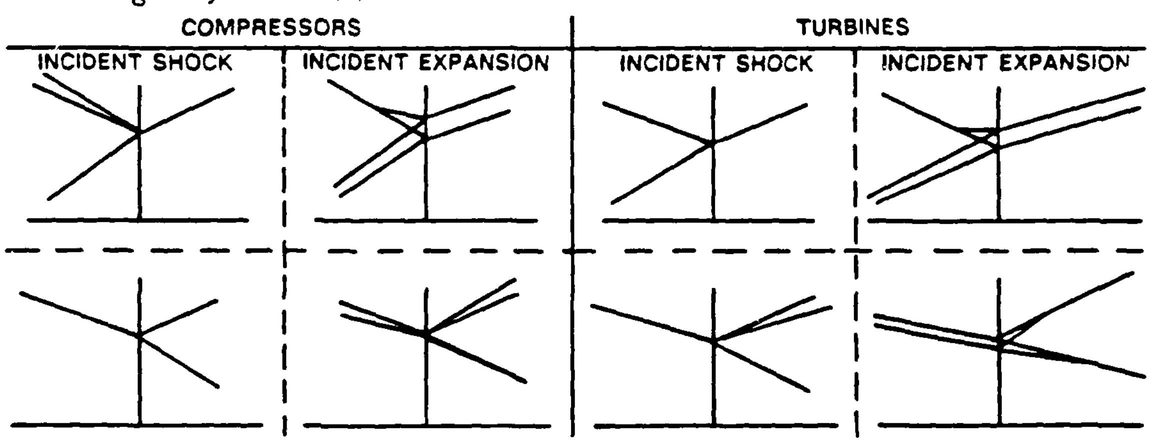
In order to make the problem manageable, each component is treated as an actuator disk (7), i.e. the axial extent of the component is assumed to be shortened into a sheet of zero thickness, across which the initial and final states are connectd by a discontinuity. This approximation is suitable as long as one examines the solution on a time scale that is large compared to the time for a sound wave to pass through the stage. During the latter time, details of the inside of the stage are important; for times large compared to this, however, the action of the stage is adequately represented by its inlet and outlet conditions, considered to take place by a discontinuous jump in a region of vanishing axial extent.

The representation of a turbomachine stage by a disk of zero axial extent across which discontinous changes in stagnation conditions occur is an excellent approximation whenever details of the flow inside the stage are not important. This approximation has been applied with great success to a number of problems. For example, the propagation of plane waves through ducts has been studied for many years, and solutions are available for the changes that occur when such waves encounter discontinuities of various kinds, such as sudden area changes, flame fronts, and other moving waves (8). Surprisingly, the solution for the case where the discontinuity is an actuator disk appears not to have been studied, although the basic equations applicable to this case are discussed in several papers (9-11).

2.1.1 Qualitative Results.

The present paper contains the solution for the waves that are generated when a pressure wave (either a shock or an expansion) arrives at an actuator disk. The disk is characterized as having a given stagnation pressure ratio, and is assumed to produce this change isentropically. The types of the reflected and transmitted waves are found (i.e., whether they are expansions or compressions), in addition to the magnitudes of the changes they produce relative to that of the incident wave. It is found that a shock wave incident from upstream on a compressor stage reflects as an expansion and is transmitted as a weakened shock. An expansion wave incident from upstream reflects from a compressor stage as a shock, and is transmitted as a weakened expansion.

These results are reversed when the wave is incident from the downstream side of a compressor stage: an incident shock reflects and is transmitted as a shock while an incident expansion produces both a transmitted and reflected expansion. These results are in agreement, both qualitatively and quantitatively, with the measurements of (1-3). The sketch below contains a qualitative summary of the transmission and reflection laws. (There are eight cases, corresponding to two values of the stagnation pressure ratio (compressor or turbine), two signs of the incident pressure rise (shock or expansion wave), and two directions in which the incident wave moves (downstream-moving or upstream-moving)). The sketch contains the wave diagrams for the various cases. The diagrams are shown in a graph whose vertical axis is the time and whose horizontal axis is axial position, in accordance with the conventional methods of one-dimensional gas dynamics (8).



In addition to clarifying the physics of such wave interactions, the present results also provide an important link to recent developments in the understanding of

surge and rotating stall (10, 11). Those developments pertain generally to engine transients of a much longer time scale (the order of the engine length divided by the speed of sound), and make use of unsteady compressor characteristics in the form of instantaneous pressure rise vs. mass flow relations. The present analysis is capable of being extended to incorporate similar characteristics, but now for individual stages rather than for the whole component. Such an application would assist in showing how surge and rotating stall develop within the component.

These analytic results can also be used to simplify the computer codes referred to above, and should be capable of extending their range of applicability. This is because the codes have difficulty in tracking the large numbers of waves that are generated; the ability to describe the locations and amplitudes of these waves analytically may enable significant improvements in the numerical modeling of these events.

2.1.2 Solution Method

Once the actuator-disk approximation is made, the solution of the problem can be carried out directly using the classic techniques of wave-diagram construction (8). The Riemann invariants which form the basis of the solution are the same as those for flow in a duct, and need only be modified for the transitions across the actuator sheet. If one were interested in details of the flow within an engine component, the analysis would have to be generalized to include the effects of angular velocity changes within the component, which would then be referred to as an 'actuator duct'. A brief discussion of these levels of approximation can be found in (7) and (9).

For the actuator-disk, one-dimensional-duct-flow model, the flow can be charted in an axial distance (x) vs time (t) diagram (see Figure 1). (Here the flow quantities are constant in various zones, which are separated by moving waves.) Thus, the vertical axis represents the actuator disk and the time axis and the lines separating regions 1 and 2 or 4 and 5 are the incident and transmitted shock waves. The broken line between regions 5 and 6 is the interface which separates gas that has or has not been affected by the reflected wave that separates regions 2 and 3. The reflected wave is shown as a shock in Figure 1; the results below will show that it is actually an *expansion* wave. Particle paths are shown as the short dashed lines.

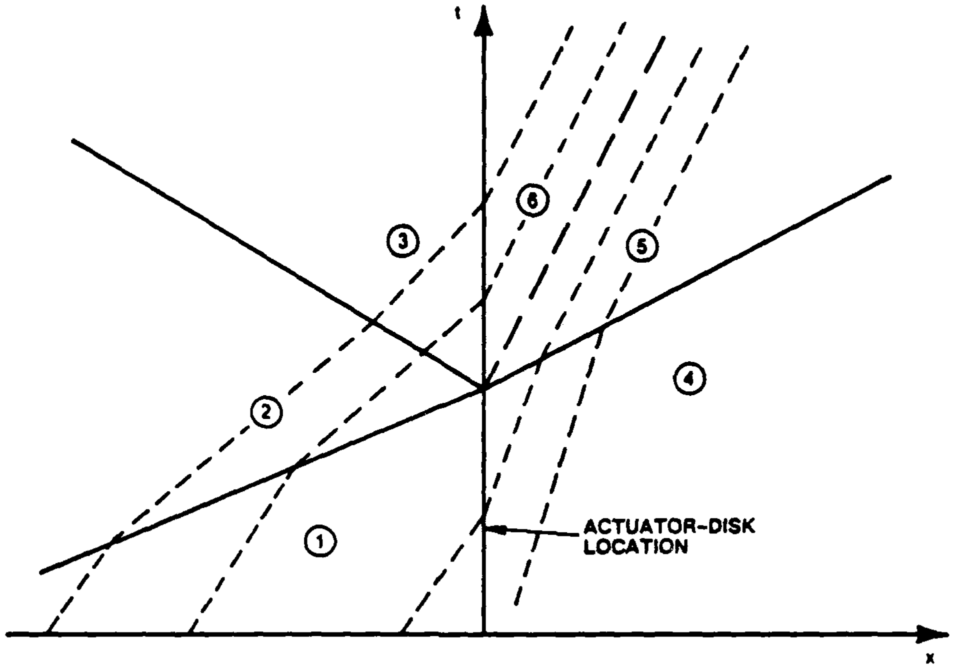


Figure 1. Wave diagram for a downstream-propagating wave.

The actuator disk is assumed to have a given constant stagnation-pressure ratio across it, and to achieve its compression isentropically, i.e.,

$$P_{0B} = \pi P_{0A}$$
$$\tau = \frac{T_{0B}}{T_{0A}} = (\pi)^{\frac{\gamma-1}{\gamma}} \tag{1}$$

where subscripts A and B denote conditions ahead of and behind the disk, P_{oi} and T_{oi} are the stagnation pressure and temperature in region i, γ is the ratio of specific heats (assumed constant), and π is a given constant. A second condition across the actuator disk is that the mass flow be constant, i.e.

$$\rho_A u_A = \rho_B u_B \tag{2}$$

where ρ and u denote the density and axial velocity, respectively.

The solution procedure is as follows:

1) The compression ratio π and specific-heat ratio γ are chosen; the initial inlet Mach number M_1 is chosen and the strength of the incident wave (as given by the pressure ratio p_2/p_1) is chosen. Conditions across the incident wave are given by either the shock-wave or expansion-wave relations. For the shock-wave case, these are:

$$\left(\frac{u_{s_{AB}} - u_A}{a_A}\right)^2 = \frac{(\gamma+1)(p_B/p_A) + (\gamma-1)}{2\gamma} \tag{3}$$

$$\left(\frac{u_{s_{AB}} - u_B}{a_B}\right)^2 = \frac{(\gamma-1)\,(p_B/p_A) + (\gamma-1)}{(2\gamma)(p_B/p_A)} \tag{4}$$

$$\frac{\rho_B}{\rho_A} = \frac{(\gamma+1)\,(p_B/p_A) + (\gamma-1)}{(\gamma-1)\,(p_B/p_A) + (\gamma+1)}\,; \quad \frac{T_B}{T_A} = \frac{p_B}{p_A}\,\frac{\rho_A}{\rho_B} \tag{5}$$

$$\frac{p_{0B}}{p_{0A}} = \left(\frac{p_B}{p_A}\right)^{-\frac{1}{\gamma-1}}\left(\frac{\rho_B}{\rho_A}\right)^{\frac{\gamma}{\gamma-1}} \tag{6}$$

where $u_{s_{AB}}$ denotes the velocity of the shock wave connecting regions A and B. The first of these gives the shock velocity, the second the particle velocity.

For the case of an expansion wave incident from the upstream side, the conditions behind the wave are found from the constancy of the Riemann invariant Q across the wave:

$$Q = \frac{2}{\gamma-1} a_B - u_B = \frac{2}{\gamma-1} a_A - u_A \tag{7}$$

where the sound speed a can be found from the isentropic relation

$$\frac{a_B}{a_A} = \left(\frac{p_B}{p_A}\right)^{\frac{\gamma-1}{2\gamma}} \tag{8}$$

The remaining equations are then:

$$u_B / a_A = \frac{2}{\gamma-1} \left[\left(\frac{p_B}{p_A}\right)^{\frac{\gamma-1}{2\gamma}} - 1\right] + u_A / a_A \tag{9}$$

$$\rho_B/\rho_A = \left(p_B/p_A\right)^{\frac{1}{\gamma}} \; ; \; T_B/T_A = \frac{p_B/p_A}{\rho_B/\rho_A} \; ; \; M_B = \left(\frac{u_B}{a_A}\right)\left(\frac{a_B}{a_A}\right)^{-1} \tag{10}$$

The leading edge of the expansion wave travels at the speed:

$$dx/dt = u_A + a_A \tag{11}$$

while its trailing edge travels at the speed $u_B + a_B$.

2) Conditions behind the actuator disk are found from

$$c_p T_B + \frac{1}{2} u_B^2 = \tau \left[c_p T_A + \frac{1}{2} u_A^2\right] \tag{12}$$

$$\rho_B u_B = \rho_A u_A$$

By using the isentropic relation:

$$T_B/T_A = \left(\rho_B/\rho_A\right)^{\gamma-1} = \left(u_A/u_B\right)^{\gamma-1} \qquad (13)$$

these can be put into the single equation

$$u_B/u_A = \left\{ \frac{\tau\left[1 + \frac{2}{(\gamma-1)\,M_A^2}\right] - \left(u_B/u_A\right)^2}{\frac{2}{(\gamma-1)\,M_A^2}} \right\}^{-\frac{1}{\gamma-1}}, \qquad (14)$$

This equation can be solved by successive substitutions, using $u_B/u_A = 0$ as an initial guess.

It should be noted that the designations "ahead of" and "behind" are to be interpreted for the actuator disk as referring to a pair of points which lie on the upstream and downstream sides of the disk, at the same instant of time on the wave diagram. When these designations are applied to moving shocks and expansions, they are to be interpreted as denoting points which are at a fixed spatial position and are, respectively, at times less than and greater than the time at which the wave passes the given position.

3) Next, values of p_5/p_4 and p_3/p_2 are chosen, and these are adjusted (by an iteration process described below) until values are found such that $p_6 = p_5$ and $u_6 = u_5$. The relations between conditions in regions 5 and 4 are given by either the shock- or expansion-wave relations above, with A=4 and B=5.

The transition 2-3 may be a reflected shock, in which case the shock relations are used, with A=2, B=3. Because this wave is moving to the left, care must be taken to choose the appropriate signs; the positive square roots are taken, and the equations for the shock and particle velocity are written as:

$$\frac{u_A - u_{S\ AB}}{a_A} = + \sqrt{\frac{(\gamma+1)\ (p_B/p_A) + (\gamma-1)}{2\gamma}} \tag{15}$$

$$\frac{u_B - u_{S\ AB}}{a_B} = + \sqrt{\frac{(\gamma-1)\ (p_B/p_A) + (\gamma+1)}{2\gamma\ (p_B/p_A)}} \tag{16}$$

In both of these expressions the quantity $u_{S\ AB}$ is a negative number.

If the transition 2-3 is a (left-running) expansion wave, the solution is given by the constancy of the Riemann invariant P :

$$P = \frac{2}{\gamma-1}\ a_A + u_A = \frac{2}{\gamma-1}\ a_B + u_B\ ;\ a_B/a_A = \left(\frac{p_B}{p_A}\right)^{\frac{\gamma-1}{2\gamma}} \tag{17}$$

This can be solved explicitly as

$$\frac{u_B}{u_A} = \frac{2}{\gamma-1}\left(1 - \frac{a_B}{a_A}\right) + \frac{u_A}{a_A} \tag{18}$$

The speed of this expansion wave varies between the limits:

$$\frac{d\chi}{dt} = u_A - a_A \quad \text{and} \quad \frac{d\chi}{dt} = u_B - a_B \tag{19}$$

The iteration process used to update the values of p_5/p_4 and p_3/p_2 is based on a straight-line extrapolation: the graphs of p_5 vs u_5 and p_6 vs u_6 found for various values of p_5/p_4 and p_3/p_2 are nearly straight lines. Thus, their point of intersection can be readily calculated, and the corresponding values of p_5/p_4 and p_3/p_2

used for the subsequent iteration. These iterations were terminated when successive approximations differed by less than 10^{-4}.

For cases where the incident wave comes from downstream, a few modifications to these relations are required. Figure 2 shows the wave diagram, which now has seven zones. As before, M_1, π and γ are given, and the relation across the actuator disk between regions 1 and 4 is given by equation 14, with A=1, B=4.

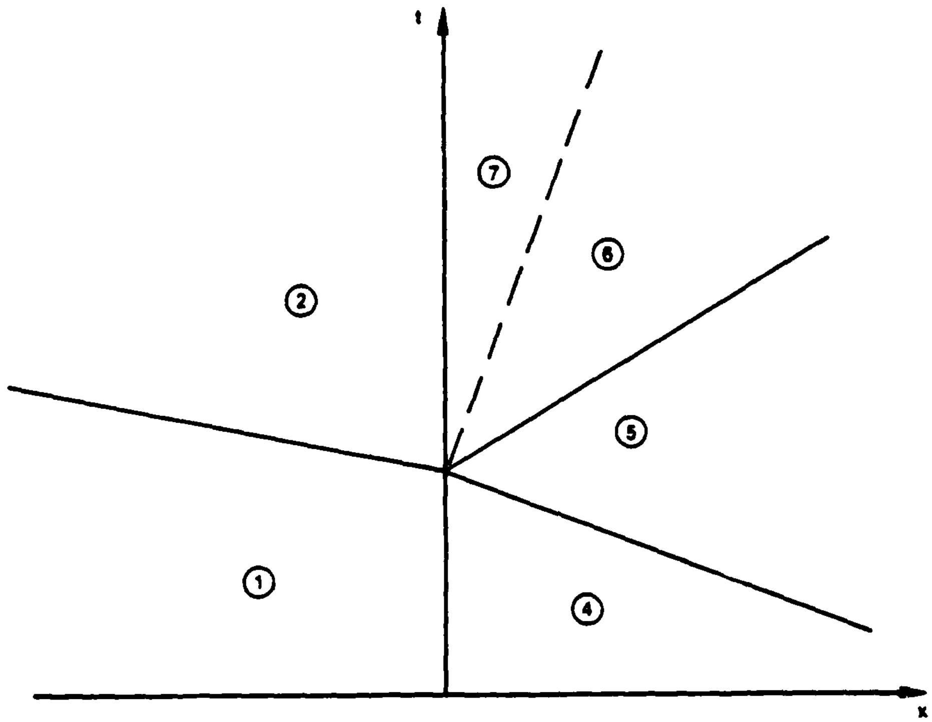


Figure 2. Wave diagram for an upstream-propagating wave.

The incident pressure ratio p_5/p_4 is also given; if it is greater than 1.0, equations 3 and 4 are used, corresponding to an upstream-moving shock; if it is less than 1.0, equations 15 and 16 are used, for an upstream-moving expansion wave.

The transition 5-6 uses the relations for a right-moving shock (equations 3-4) or expansion (equations 7-8), while that between regions 1 and 2 uses the relations for a left-moving shock (equations 15-16) or expansion (equation 17). Regions 2 and 7 are connected by the actuator-disk relation, equation 4, with A=2, B=7.

An iteration process similar to that used for waves incident from the upstream side is now used: a series of wave strengths p_6/p_5 produces a graph of u_6 versus p_6, while a series of strengths p_2/p_1 produces a graph of u_7 versus p_7. Linear

fits to these curves are used to find improved values of p_6 and p_2 (the latter requiring a further linear fit to the variation of p_2 versus p_7).

Fortran programs incorporating these formulas are given in Appendices A and B.

2.2 NUMERICAL RESULTS.

Consider first the case of a shock wave incident from the upstream side of a compressor stage. Tables 1-4 present typical results: the first one shown is a baseline case, chosen for its similarity to the first-stage-fan data of (1). The subsequent tables then show the effects of changing, one at a time, the parameters, π, M_1 and p_2/p_1.

Table 1. Numerical solution for $\pi = 2$, $M_1 = 0.2$, $\gamma = 1.4$, $p_2/p_1 = 1.1$

i =	2	3	4	5	6
p_i/p_1	1.1	1.0494	2.0397	2.1950	2.1951
ρ_i/ρ_1	1.0704	1.0350	1.6639	1.7534	1.7534
T_i/T_1	1.0276	1.0139	1.2259	1.2519	1.2519
u_i/a_1	.2636	.3025	.1202	.1786	.1786
M_i	.2649	.3004	.1086	.1596	.1596
a_i/a_1	1.0137	1.0069	1.1072	1.1189	1.1189
p_{oi}/p_{o1}	.9999	.9999	2.0	1.9999	1.9998

$\dfrac{(\rho u)_{3,6}}{(\rho u)_{1,2}} = 1.566$

Table 2. Numerical solution for $\Pi = 4$, $M_1 = 0.2$, $\gamma = 1.4$, $p_2/p_1 = 1.1$

i =	2	3	4	5	6
p_i/p_1	1.1	1.0061	4.1029	4.3170	4.3170
i/ 1	1.0704	1.0043	2.7411	2.8425	2.8424
T_i/T_1	1.0276	1.0018	1.4968	1.5187	1.5188
u_i/a_1	.2686	.3327	.0730	.1176	.1176
M_i	.2649	.3324	.0596	.0954	.0954
a_i/a_1	1.0137	1.0009	1.2234	1.2324	1.2324
p_{oi}/p_{o1}	.9999	.9999	4.0	3.9999	3.9996

$\frac{(\rho u)_{3,6}}{(\rho u)_{1,2}} = 1.671$

Table 3. Numerical solution for $\Pi = 2$, $M_1 = 0.2$, $\gamma = 1.4$, $p_2/p_1 = 1.5$

i =	2	3	4	5	6
p_i/p_1	1.5	.9914	2.0397	2.8149	2.8149
i/ 1	1.3333	.9919	1.6639	2.0923	2.0923
T_i/T_1	1.1250	.9995	1.2259	1.3454	1.3467
u_i/a_1	.4988	.8035	.1202	.3813	.3813
M_i	.4703	.8037	.1086	.3287	.3285
a_i/a_1	1.0607	.9997	1.1072	1.1599	1.1605
p_{oi}/p_{o1}	.9932	.9932	2.0	1.9932	1.9865

$\frac{(\rho u)_{3,6}}{(\rho u)_{1,2}} = 3.985$

Table 4. Numerical solution for $\Pi = 2$, $M_1 = 0.4$, $\gamma = 1.4$, $p_2/p_1 = 1.1$

i =	2	3	4	5	6
p_i/p_1	1.1	1.0128	2.1680	2.3374	2.3374
i/ 1	1.0704	1.0091	1.7380	1.8339	1.8339
T_i/T_1	1.0276	1.0037	1.2474	1.2746	1.2746
u_i/a_1	.4686	.5280	.2302	.2905	.2905
M_i	.4622	.5270	.2061	.2573	.2573
a_i/a_1	1.0137	1.0018	1.1169	1.1290	1.1290
p_{oi}/p_{o1}	.9999	.9999	2.0	1.9999	1.9998

$\frac{(\rho u)_{3,6}}{(\rho u)_{1,2}} = 1.332$

Several items are worthy of note:

1) The reflected wave is an expansion. It might have been expected that the wave reflected from a high-solidity compressor face would be a shock wave, but these results show clearly that it is an expansion wave, for the whole range of cases studied. This finding is also confirmed by the pressure/time traces of (1).

2) The "transmission coefficient", defined as

$$T_{\pi} \equiv \frac{p_5 - p_4}{p_2 - p_1} \tag{20}$$

lies in the range 0.5 to 0.8 of π, favoring the 0.8 π value when π itself is 2.0. This result compares favorably with the values presented in (1) (see Figs. 19 and 20).

3) The arrival of the incident shock causes a very large change in mass flow rate - on the order of a factor of 1.5 - despite the fact that the incident wave is a relatively weak shock.

4) Other calculations, not shown here, reveal that these qualitative conclusions are not sensitive to variations in the specific-heat ratio.

The agreement between the predicted and observed values of the transmission coefficient is encouraging, while the indication that the reflected wave is an expansion is in qualitative agreement with the results of (1) and (3). The conditions of the latter experiment contain other factors that affect the pressure signals; these are discussed at greater length below. Overall, these results are a useful guide in interpreting the response of individual stages.

The large mass flow perturbation raises the question whether the actuator-disk stagnation-pressure ratio can properly be regarded as constant. While it is true that the engine RPM does not change on the time scale of these wave transits, nevertheless the unsteady compressor characteristics used, for example, in the work of Moore and Greitzer(12,13) indicate that the quantity π should be allowed to vary with $\rho_3 u_3$. Very little is known at present about the variation of pressure with mass flow on the relatively short time scale of a shock passage through a stage, and it is not clear that the longer-time variations used in the rotating-stall work cited above

are accurate for the present problem. Further investigation of the detailed mass-flow/pressure variations during wave passage through a single stage is needed to resolve this question.

Results of further calculations are given in Figures 3-4 for shocks and expansions which are incident both from upstream and downstream. These figures confirm the general transmission and reflection laws mentioned in the Introduction.

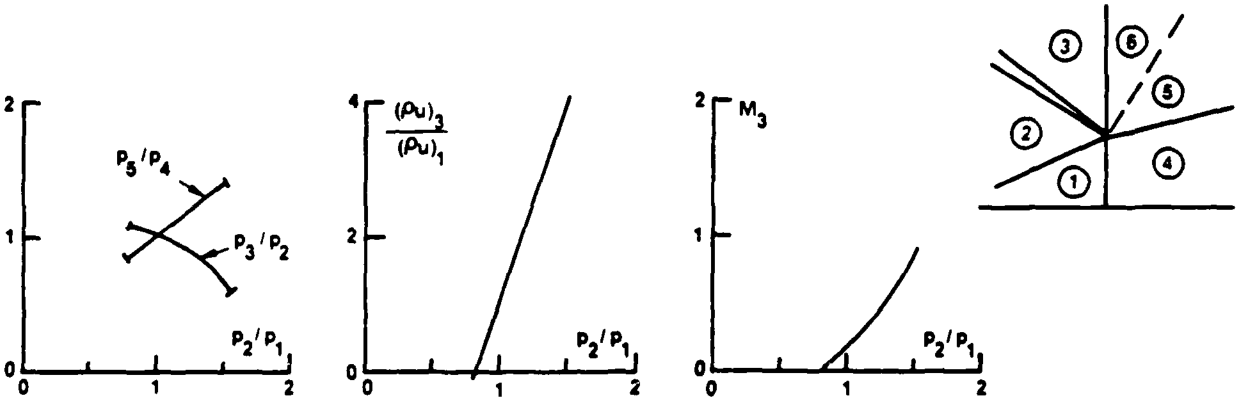


Figure 3. Actuator-disk solutions for $\pi = 2.0$, $M_1 = 0.2$, $\gamma = 1.4$ (Downstream-propagating waves).

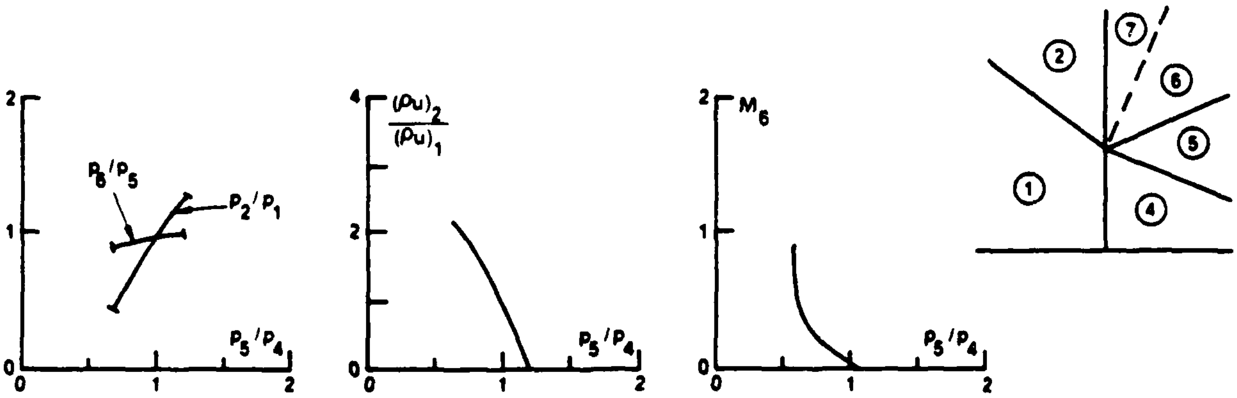


Figure 4. Actuator-disk solutions for $\pi = 2.0$, $M_1 = 0.2$, $\gamma = 1.4$ (Upstream-propagating waves).

Similar results have been found for the transmission and reflection of sound waves in turbomachinery components (14-15). The latter studies are somewhat more complex, in that they consider sinusoidal waves and allow for wavefront propagation

in directions other than parallel to the axis of rotation. For the case of axial propagation, these papers reach the same conclusions about transmission and reflection as those derived above. The present results show that there is no qualitative change in the transmission and reflection laws for finite wave amplitudes.

There are limits to the validity of the present solution, however. The curves in Figures 3-4 stop at finite limits; for example, in the case of waves incident from the upstream side, a sufficiently strong reflected shock will stop the flow ($M_3 = 0$), while a sufficiently strong reflected expansion wave will choke the flow ($M_3 = 1$). Beyond these limits, there is no solution of the type being considered here. It is interesting to note that these limits are reached for fairly moderate values of the incident wave strength; for example, an incident expansion wave with a pressure drop of as little as twenty percent of the initial pressure is sufficient to stop the flow.

SECTION 3
ANALYTICAL SOLUTIONS FOR WEAK WAVES

The incident-wave strengths of interest in many applications have pressure ratios close to one; thus it is useful to examine the analytic forms that the full equations reduce to in this limit.

Consider first the case where the initial disturbance is incident from upstream: p_{02}/p_{01}, M_1, and γ are given. Let the (given) incident wave strength be denoted by

$$\frac{p_2}{p_1} = 1 + \epsilon \quad , \quad \epsilon << 1 \tag{21}$$

It can be expected that the solution will have the form

$$\frac{p_3}{p_2} = 1 + \delta \, , \quad \frac{p_5}{p_4} = 1 + \nu \; ; \quad \delta \left(\epsilon\right) \ll 1 \, , \; \nu \left(\epsilon\right) \ll 1 \tag{22}$$

Where the dependence of δ and ν on ϵ is to be found from the usual matching condition $p_6 = p_5$, $u_6 = u_5$.

The quantities ϵ , δ , and ν can all be positive or negative, with no change in the functional form of the equations, since weak shocks and expansions are indistinguishable to the third order in small disturbances.

The desired equations are found by expanding the exact relations in Taylor series (with the exception of the transition across the actuator disk - see below). In these expansions, the Mach numbers are not assumed to be small, although their numerical contributions will in some cases be negligible.

The pressures in regions 5 and 6, which are required to be equal, are

$$\frac{p_5}{p_1} = \frac{p_5}{p_4}\,\frac{p_4}{p_1} = \frac{p_4}{p_1} \left(1 + \nu\right) \tag{23}$$

$$\frac{p_6}{p_1} = \frac{p_6}{p_3}\,\frac{p_3}{p_2}\,\frac{p_2}{p_1} = \frac{p_6}{p_3}\left(1 + \delta + \epsilon + ----\right) \tag{24}$$

Thus

$$\frac{p_4}{p_1}\left(1 + \gamma\right) = \frac{p_6}{p_3}\left(1 + \delta + \epsilon\right) \tag{25}$$

The particle velocities in regions 5 and 6 can be written as:

$$\frac{u_5}{a_1} = \frac{u_5}{u_4}\,\frac{u_4}{u_1}\,\frac{u_1}{a_1} = M_1\,\frac{u_4}{u_1}\,\frac{u_5}{u_4}$$

$$= M_1\,\frac{u_4}{u_1}\left(1 + \frac{\nu}{\gamma M_4}\right) \tag{26}$$

$$\frac{u_6}{a_1} = \frac{u_6}{u_3}\,\frac{u_3}{a_1} = \frac{u_6}{u_3}\left(M_1 + \frac{\epsilon - \delta}{\gamma} + ----\right) \tag{27}$$

Equating these two gives

$$\frac{u_4}{u_1}\left(1 + \frac{\nu}{\gamma M_4}\right) = \frac{u_6}{u_3}\left(1 + \frac{\epsilon - \delta}{\gamma M_1}\right) \tag{28}$$

A simultaneous solution of equations 25 and 28 gives:

$$\delta = \frac{\frac{p_4}{p_1}\left(\frac{u_6}{u_3} - \frac{u_4}{u_1}\right) - \frac{u_4}{u_1}\left(\frac{p_6}{p_3} - \frac{p_4}{p_1}\right)\frac{1}{\gamma M_4} + \epsilon\left[\frac{1}{\gamma M_1}\frac{p_4}{p_1}\frac{u_6}{u_3} - \frac{1}{\gamma M_4}\frac{p_6}{p_3}\frac{u_4}{u_1}\right]}{\frac{1}{\gamma M_1}\frac{p_4}{p_1}\frac{u_6}{u_3} + \frac{1}{\gamma M_4}\frac{p_6}{p_3}\frac{u_4}{u_1}} \tag{29}$$

$$\nu = \frac{\frac{1}{\gamma M_1}\frac{u_6}{u_3}\left(\frac{p_6}{p_3} - \frac{p_4}{p_1}\right) + \frac{p_6}{p_3}\left(\frac{u_6}{u_3} - \frac{u_4}{u_1}\right) + \epsilon\left[\frac{1}{\gamma M_1}\frac{u_6}{u_3}\frac{p_6}{p_3} + \frac{1}{\gamma M_1}\frac{p_6}{p_3}\frac{u_6}{u_3}\right]}{\frac{1}{\gamma M_1}\frac{p_4}{p_1}\frac{u_6}{u_3} + \frac{1}{\gamma M_4}\frac{p_6}{p_3}\frac{u_4}{u_1}} \tag{30}$$

These equations cannot be solved directly, because the pressure and velocity ratios across regions 3 and 6 require knowledge of M_3, which in turn depends on ϵ and δ:

$$M_3 = M_1 + \frac{\epsilon - \delta}{\gamma} - \frac{\gamma - 1}{2\gamma} M_1 \left(\delta + \epsilon\right) + \text{-----} \tag{31}$$

In addition, the fact that the actuator-disk transition requires the solution of an implicit equation prevents the derivation of an explicit solution valid for arbitrary stagnation pressure ratios.

However, for the case where M_1 is small (and of the same order as ϵ) a fully analytic solution is possible. In this limit,

$$\delta = -\frac{M_1 - M_4}{M_1 + M_4}\epsilon , \quad \nu = \frac{2M_4}{M_1 + M_4}\epsilon \tag{32}$$

Moreover, M_1 and M_4 are related in this limit, so that the solution takes the simple form

$$M_4 = M_1 \pi^{-\frac{\gamma+1}{2\gamma}} ; \quad \frac{\delta}{\epsilon} = -\frac{1 - \pi^{-\frac{\gamma+1}{2\gamma}}}{1 + \pi^{-\frac{\gamma+1}{2\gamma}}} ; \quad \frac{\nu}{\epsilon} = \frac{2\pi^{-\frac{\gamma+1}{2\gamma}}}{1 + \pi^{-\frac{\gamma+1}{2\gamma}}} \tag{33}$$

These relations contain, qualitatively, the results of the previous section. For a compressor, for example, M_4 is less than M_1; thus shocks reflect as expansions and

vice versa, while the transmitted wave is of the same type as the incident wave, with reduced amplitude. For a turbine, M_4 is greater than M_1; thus incident shocks and expansions are reflected as shocks and expansions of reduced amplitude, and are transmitted as shocks and expansion of increased amplitude.

The derivation of these results is based on approximating the solution of the (implicit) actuator-disk equation for small values of the upstream Mach number:

$$\left(\frac{u_B}{u_A}\right)^{-(\gamma-1)} = \tau\left[1+\frac{\gamma-1}{2}M_A^2\right] - \frac{\gamma-1}{2}M_A^2\left(\frac{u_B}{u_A}\right)^2$$
$$= \tau\left[1+\frac{\gamma-1}{2}M_A^2\right] + 0\left(M_A\right)^2 \qquad (34)$$

Thus
$$\frac{u_B}{u_A} = \tau^{-\frac{1}{\gamma-1}}\left[1+0\left(M_A^2\right)\right] \qquad (35)$$

By substituting this back on the right-hand side of the original equation, it is found that

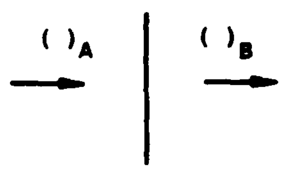
$$\frac{u_B}{u_A} = \tau^{-\frac{1}{\gamma-1}}\left\{1-\frac{M_A^2}{2}\left(1-\tau^{-\frac{\gamma+1}{\gamma-1}}\right)\right\} + 0\left(M_A^4\right) \qquad (36)$$

For a compressor, this equation is valid within three percent over the range $0 \leq M_A \leq 1$ for $\gamma = 1.4$ and $\pi = 2$ (see Figure 5). The approximation is not as accurate for a turbine, where $u_B/u_A > 1$; the results in Figure 6 show that it is accurate to a few percent over about two thirds of the allowable range of M_A. This corresponds to the range for which the Mach number downstream of the disk stays less than about 0.75.

This approximate solution can now be used to simplify the small-disturbance solution (eqs. 29 and 30). The pressure ratio is given by

$$\frac{p_B}{p_A} = \left(\frac{\rho_B}{\rho_A}\right)^{\gamma} = \left(\frac{u_A}{u_B}\right)^{\gamma} = \pi\left\{1+\frac{\gamma E}{2}M_A^2\right\} + 0\left(M_A^4\right) \qquad (37)$$
$$E \equiv 1+\tau^{-\frac{\gamma+1}{\gamma-1}} = 1-\pi^{\frac{\gamma+1}{\gamma}}$$

ACTUATOR-DISK SOLUTIONS



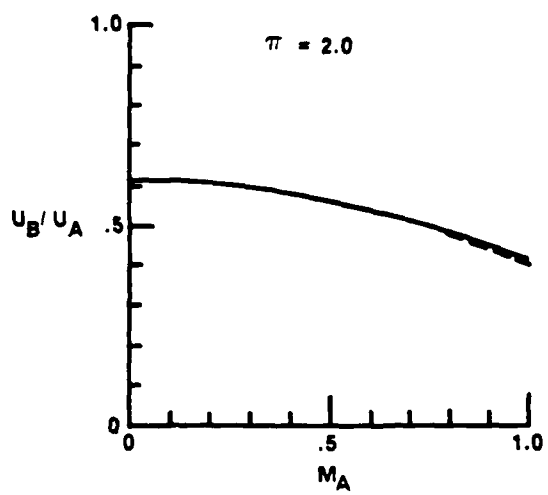


Figure 5. Actuator-disk solution for π = 2.0.

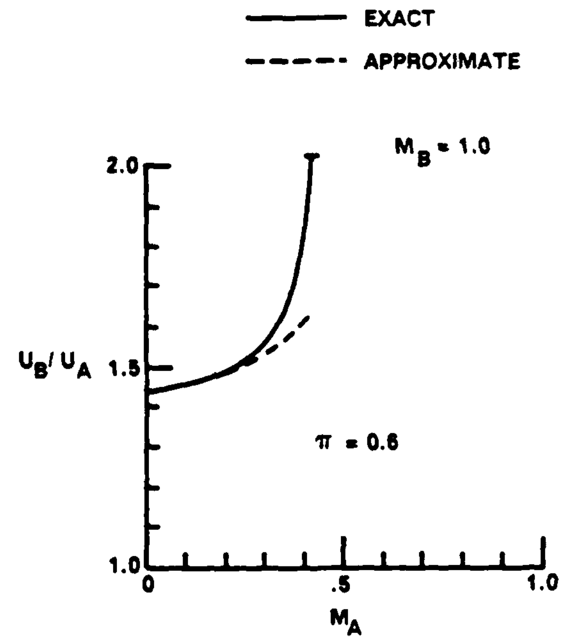


Figure 6. Actuator-disk solution for π = 0.6.

When eqs. 29 and 30 are expanded using these equations, it is found that all of the pressure- and velocity-ratio terms cancel, leaving a remainder of order $M_1^2\epsilon$, which is of higher order than the terms retained. A number of intermediate solutions can also be found, if certain combinations of terms are retained and the relative orders of M_1 and ϵ are ignored. Given the simplicity of the numerical solutions themselves, it is recommended that they be used where accuracy is considered important, and that the analytic solutions be used where qualitative information is the primary requirement.

For the case where the incident wave comes from the downstream side, a similar analysis leads to

$$\beta = - \frac{M_4 - M_1}{M_1 + M_4} \alpha$$

$$\mu = \frac{2M_1}{M_1 + M_4} \alpha \tag{38}$$

where

$$\frac{P_5}{P_4} = 1 + \alpha \; ; \quad \frac{P_6}{P_5} = 1 + \beta \; ; \quad \frac{P_2}{P_1} = 1 + \mu \; ; \; \alpha \text{ given} \tag{39}$$

These approximate formulas (eqs. 32 and 38) confirm the transmission and reflection laws summarized in Section 2.

SECTION 4
MULTIPLE REFLECTIONS BY A PAIR OF ACTUATOR DISKS

As will be seen below, the above analysis is in general agreement with the experiments of (1-3), in that the pressure measured upstream of the compressor shows a reflected expansion, following the passage of an incident shock. The detailed waveform of the reflection is not a step function for several reasons, among them the fact that the incident wave is not a step function, as well as the fact that the reflected waveform is affected by multiple reflections from the various stages of the compressor.

In order to shed some light on the latter process, calculations have been done for two pairs of actuator disks: the first pair consists of two stages of compression, while the second consists of a compressor followed by a turbine.

For the first pair, both disks were taken to have stagnation-pressure ratios of 2.0 and an incident shock strength $p_2/p_1 = 1.2$ was used with an initial flow Mach number of 0.2 and specific-heat ratio $\gamma = 1.4$ The main results are shown in Fig. 7, which contains pressure and mass-flow data in the various regions of the x,t diagram.

Before discussing these results, two observations are necessary: the first is that all entropy discontinuities have been ignored. In general, the transmitted waves are followed by interfaces across which the pressure and velocity are matched, but there is a discontinuity in entropy (or, equivalently, in local stagnation conditions) across the interface. Because the wave amplitudes and flow Mach numbers studied in these cases are small, it is legitimate to neglect the waves that would normally be generated by interactions with the interface, and the validity of this assumption is borne out by the fact that the stagnation quantities calculated here are essentially constant throughout the region of multiple reflections.

A second observation is that the wave speeds have all been taken as equal to $\pm a$ in Figures 7 to 9 to simplify the presentation. The actual wave speeds vary by $\pm 20\%$ about this baseline value, and expansion waves diverge as they propagate, but these details are suppressed in the presentation.

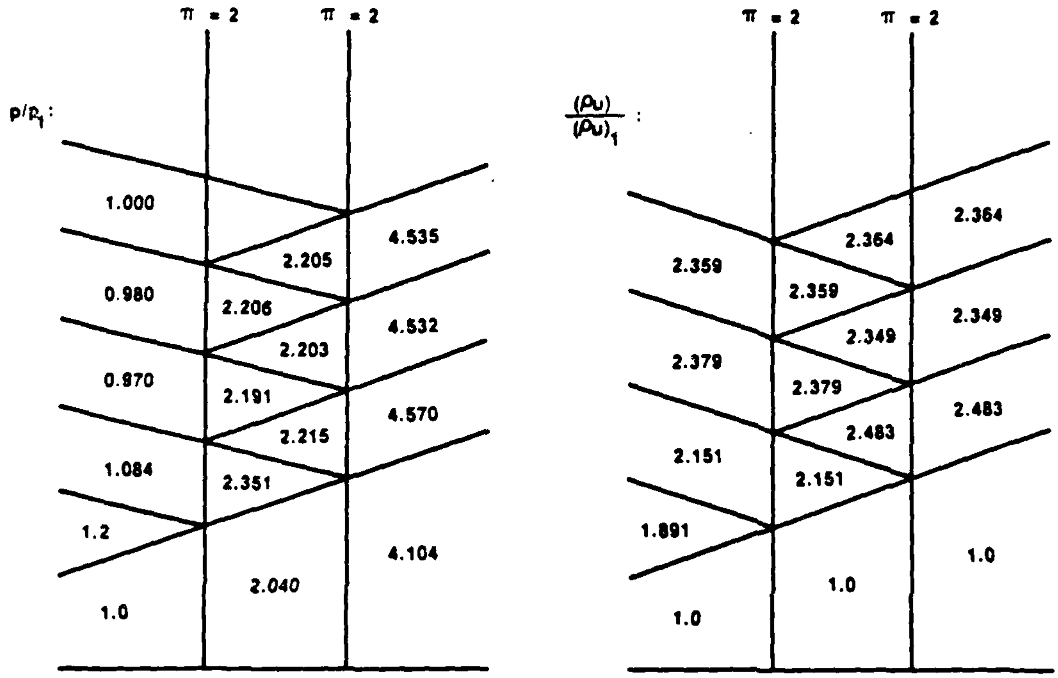


Figure 7. Pressure and massflow values during wave reflections by two compressor actuator disks; $M_1 = 0.2$, $\gamma = 1.4$.

The pressures shown in Figure 7 for the two stage compressor case indicate that the inlet pressure returns to its original level after about four reflected waves, and undergoes a slight undershoot during its return. The mass-flow data show a monotonic increase to the post-disturbance value on the upstream side of the compressor, while the values downstream overshoot at first and then subside. After about four reflections, the mass flow rate has been increased by a factor of 2.36. This is the same result (to three significant figures) as would be obtained by a single actuator disk of stagnation pressure ratio equal to 4.0. The fact that the mass flow can be sustained at this level even while the inlet pressure has returned to its pre-disturbance value assumes that the conditions far upstream remain unchanged; i.e., no waves come from upstream after the initial disturbance.

Figure 8 displays the pressure-time history that would be seen at a distance L ahead of the first disk. It bears a qualitative resemblance to the measurements of (3), and serves to emphasize that the reflected system corresponding to a step input will be a series of expansions and compressions. For a multistage compressor, this series will appear to be nearly a continuous variation.

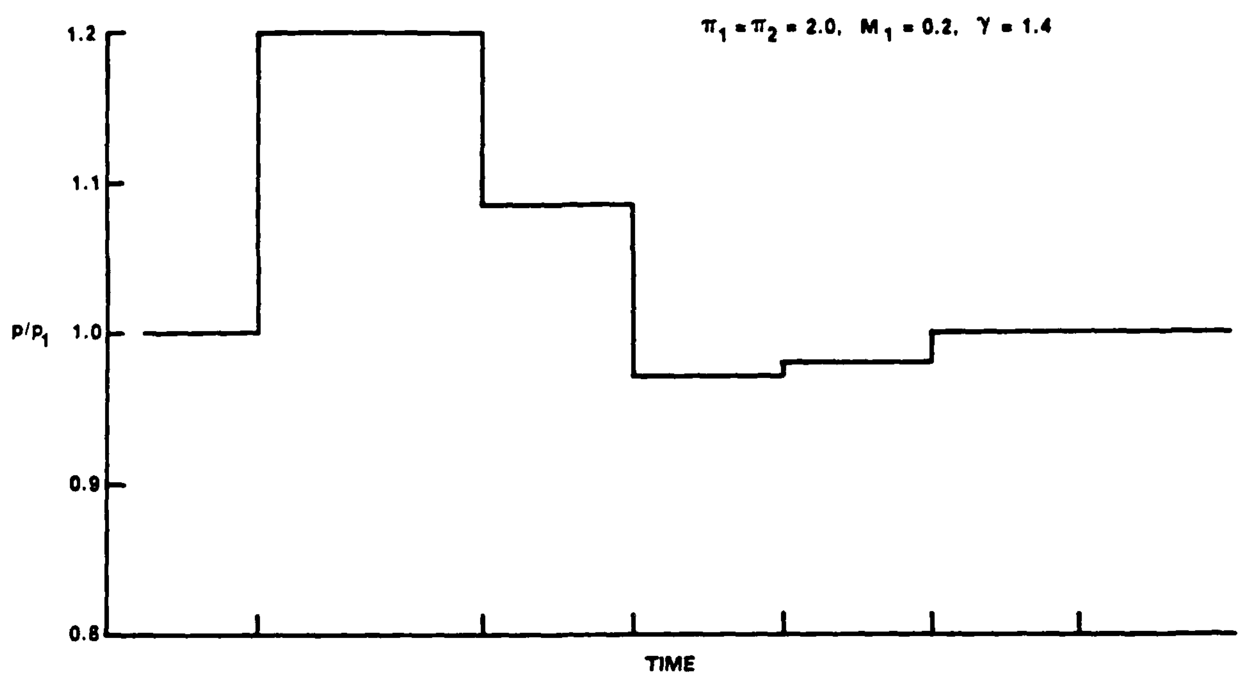


Figure 8. Static-pressure history at a station upstream of a pair of compressive actuator disks.

Results for the compressor/turbine pair of actuator disks are shown in Figure 9, using the same format as above. In this case, there is an overshoot in the inlet mass flow, while the inlet pressure falls at first and then returns to the value generated by the first incident wave. These final values are achieved after about four reflections; further waves have pressure ratios of one. The return of the pressure to the value generated by the first wave is consistent with what would be predicted by collapsing the two actuator disks into a single one: since the product of their stagnation pressure ratios is 1.0, it is as though no disks at all were present, and the long-term solution remains at the values set by the incident wave.

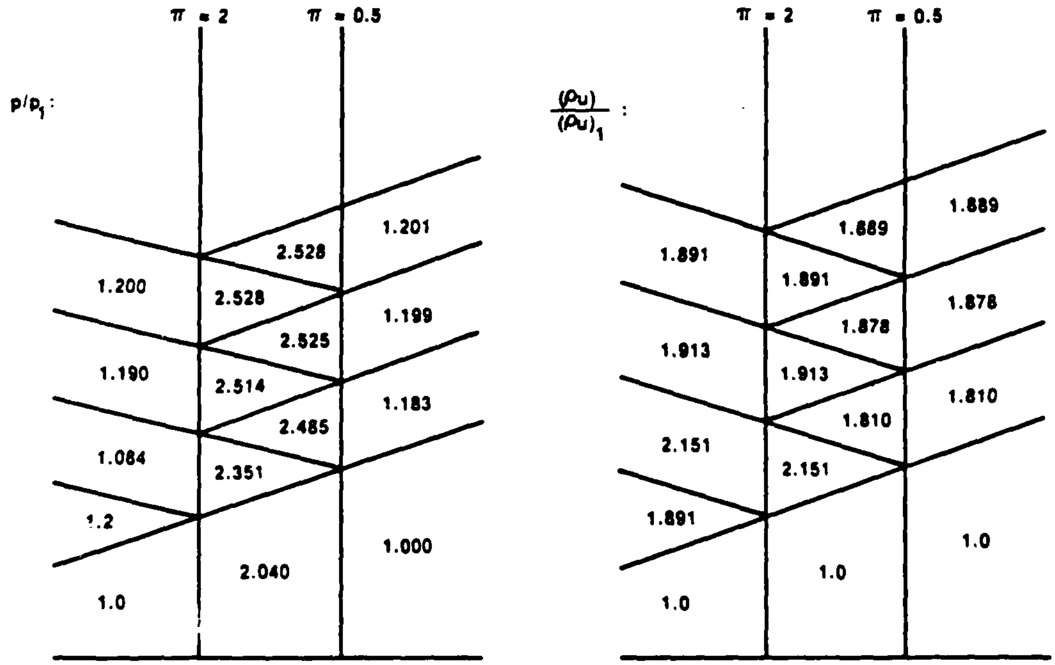


Figure 9. Pressure and massflow values during wave reflections by a compressor turbine actuator-disk pair; $M_1 = 0.2$, $\gamma = 1.4$.

4.1 COMPARISON WITH EXPERIMENT.

As noted earlier, there are many features of the full-scale experiment (1-3) that are not accounted for in the present analysis. Nevertheless, several of the qualitative features of the measurements are the same as those noted here. Figure 10 shows the pressure-time histories recorded at several stations upstream of the inlet of the TF33 engine. These data are replotted in Figure 11 so as to show the pressure waveforms, at three instants of time. Note first that at 10 milliseconds (zero time corresponds to the instant when the incident wave emerges from the shock tube) the incident wave is not a step function, but rather shows an increasing amplitude with distance from the engine inlet ($x = 0$). At $t = 20$ milliseconds, however, the pressure nearest the engine inlet has dropped, and this lower-pressure region appears to have moved through the incident pulse on the 30-millisecond trace.

The fact that the incident wave is not a step function is due to several effects; one of the primary ones is the area change at the shock-tube exit and also that at the engine inlet. It is virtually impossible to maintain a strictly flat pressure profile

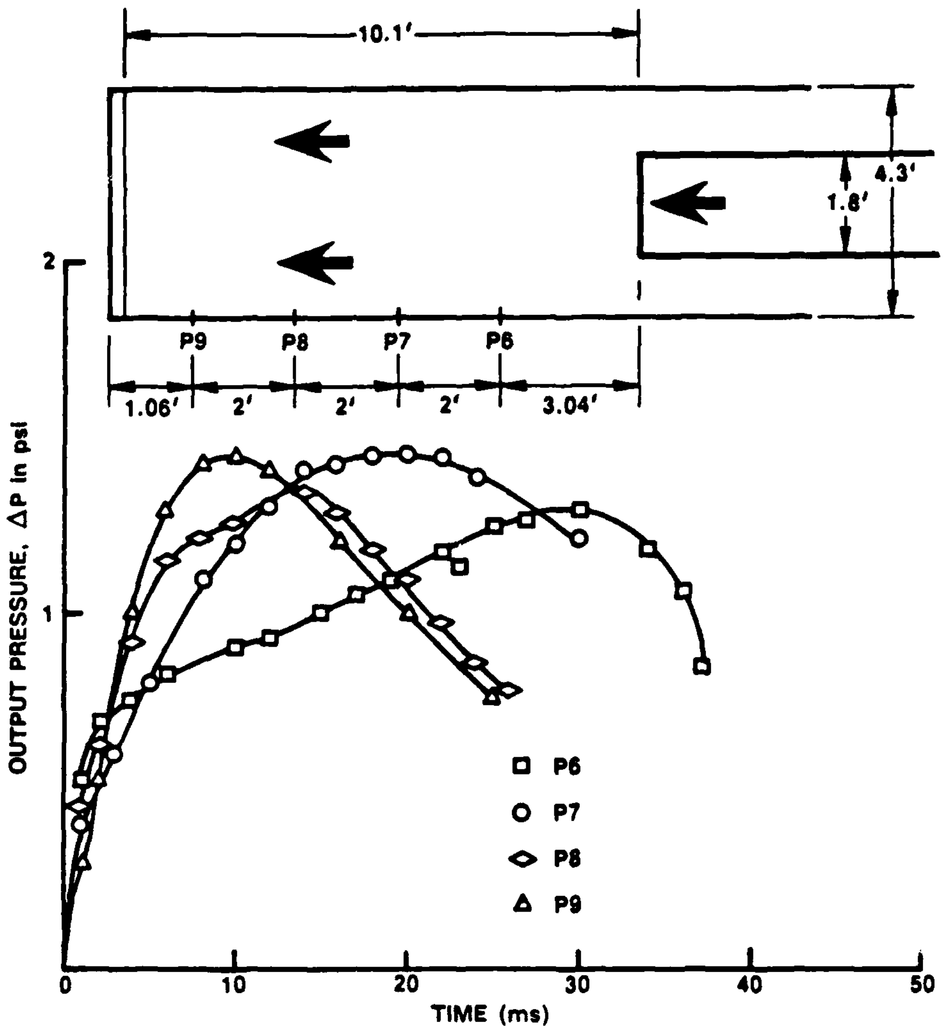


Figure 10. Pressure time histories.

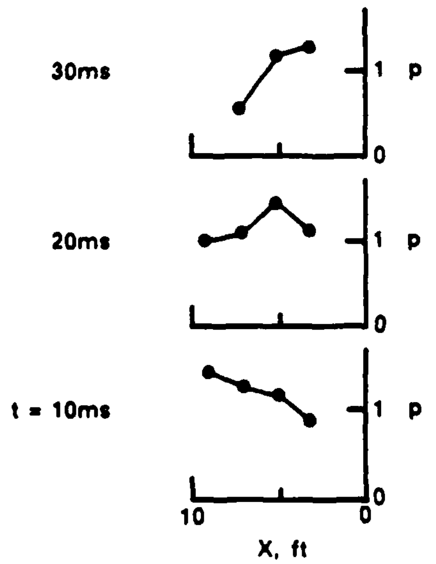


Figure 11. Pressure waveforms.

in this experiment, and a rigorous interpretation of the data requires a more detailed calculation in which the evolution of the incident waveform can be followed. Moreover, the details of the reflected waveform must take into account reflections from all of the internal stages, in order to match in complexity the experimental conditions. Thus, while the interpretation of the data shown in Figures 10 and 11 must remain qualitative, it is nevertheless encouraging to find that the overall behavior is consistent with a model which predicts that an incident compression reflects from the inlet as an expansion.

SECTION 5
CONCLUDING REMARKS

As pointed out above, numerical predictions of the phenomena measured in experiments with real, operating engines require the incorporation of many effects, multiple internal reflections being one of the most important. Unfortunately, present-day computer codes are not adequate to handle this problem. Indeed, the present research was undertaken in an effort to clarify the basic physics of wave transmission and reflection at a compressor or turbine stage. That goal has been achieved: simple reflection and transmission laws have been derived, and are qualitatively in accord with the measurements. Moreover, calculations for multi-stage machines can be done by incorporating the actuator-disk equations as a means of linking adjacent stages.

A key element is missing from current understanding of the physics, however, namely the departure of the mass-flow/pressure-ratio relation from its steady-state value. The results reported here have all been based on the assumption that the stagnation pressure ratio can be specified as a constant, independent of the mass flow rate. The results themselves show clearly that this approximation needs to be improved, since they predict mass flow changes by factors on the order of 1.5 during the passage of a wave. Excursions of this magnitude exceed the limits of many steady-state compressor or turbine maps, and call for improved modeling.

Recent studies of rotating stall and surge (12-13) have enjoyed considerable success, due in part to their use of unsteady stage-performance characteristics, in which the pressure rise Δp is taken to depend on both the flow rate ϕ and its rate of change $\dot{\phi}$:

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = F(\phi) - \tau(\phi)\dot{\phi} \qquad (40)$$

The time constant τ appearing in this expression is related to the time required to fill the plenum volume downstream of the compressor (the burner inlet, for example) and so this relation is probably not applicable to the present problem, where the mass flow changes occur on a much shorter time scale.

What is needed is a combined experimental and theoretical study, aimed at identifying the instantaneous mass flow/pressure rise relation for a single stage. The experimental apparatus to accomplish this is already available at Calspan; with only slight modifications, it would be possible to send controlled pressure waves through a single stage, which would be specially instrumented to measure the (time-dependent) mass flow and the pressure variations in the transmitted and reflected waves.

In parallel with these experiments, an analytical study would be undertaken to solve for the detailed modifications of the flow that occur when a plane wave passes through a stage. The work required amounts to a calculation of shock propagation through a pair of curved channels, whose total turning angle in the absolute frame of reference is zero. The analogous problem of shock propagation through an area change has been examined by Whitham, Chisnell and Miles (16-18) and has led to extremely simple relations for describing flows through ducts whose cross-sectional area undergoes large changes.

The clarification of the pressure-ratio/mass flow relation at the level of a single stage and on a very short time scale is the key ingredient that prevents current state-of-the-art codes from predicting wave passage through multistage components. Once it is known, these codes can be revised in such a way as to embrace the correct physics at the basic level.

Knowledge of this microscopic relation also holds the key to the relation between these short-duration disturbances and the longer-term problems of surge and rotating stall. Different time scales are involved in blast-generated transients from those of surge and stall, and it is necessary to understand the detailed mechanics of the former phenomena in order to clarify how these disturbances lead into the latter ones.

SECTION 6
RECOMMENDATIONS

In order to make further progress beyond the qualitative understanding described in this report, it is necessary to investigate the detailed relation between pressure ratio and mass flowrate during the passage of a pressure transient. This will require a study of the mechanics of pressure propagation through a rotating stage, on the short time scale given by the stage length divided by the speed of sound.

The basis for this recommendation is as follows: there are two principal mechanisms by which the ingestion of a pressure wave may damage an operating engine:

1) the overpressure may produce structural damage

2) a condition of surge or rotating stall may be induced

In the first case, the results of this report support the general results of previous measurements at Calspan: the transmission coefficients are less than one, which suggests that if the structure ahead of the compressor face can sustain the overpressure, then the remaining components in the flow path will presumably also sustain the transient.

However, there is always the possibility that certain geometries (stage lengths, for example) or incident wave forms might lead to focusing of multiply reflected waves, or that massflow amplification might produce an instantaneous choking that would send a hammber shock upstream. To resolve this question, it is necessary to do a more complete job of linking multiple stages, and this in turn requires an accurate model of the time-dependent pressure/massflow relation for each stage.

For the second mechanism described above, the recommended research is the same. Current studies of rotating stall and surge make extensive use of a time-dependent pressure/massflow relation that is based on experiments and analyses of the much longer transients associated with filling of the plenum chamber downstream of a compressor. In order to close the gap between these models and the results of the present report, it is necessary to derive the comparable relation for propagation of a pressure wave through a single stage.

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APPENDIX A

FORTRAN PROGRAM FOR DOWNSTREAM-PROPAGATING WAVES

```
C  PROGRAM WAVEB - CALCULATES THE TRANSMISSION AND REFLECTION OF
C  PRESSURE WAVES AT AN ACTUATOR DISK.
C  WAVEB CAN HANDLE INCIDENT EXPANSIONS AND SHOCKS
      OPEN(UNIT=2,FILE='WAVEOUT',STATUS='NEW')
    1 PRINT*,'INPUT STAGE PRESSURE RATIO (99. TO STOP),EM1,GAMMA'
      READ*,PR,EM1,GM
      IF(PR.EQ.99.) GO TO 30
      GMINV = 1./GM
      GM1 = GM - 1.
      GM1INV = 1./GM1
      TBGM1 = 2./GM1
      GBGM1 = GM/GM1
      GP1 = GM + 1.
      GM1BG = GM1/GM
      GM1B2 = GM1/2.
      GM1BTG = GM1BG/2.
      TAU = PR**GM1BG
      WRITE(2,100) PR,EM1,GM,TAU
  100 FORMAT(10X,'SOLUTION FOR PRESSURE WAVES INCIDENT ON AN ACTUATOR',
     *   ' DISK',/10X,'OF STAGNATION-PRESSURE RATIO =',F5.1,/10X,
     *   'INLET MACH NUMBER =',F5.2,'  SPECIFIC-HEAT RATIO =',F5.2,
     *   /10X,'STAGNATION-TEMPERATURE RATIO IS',F7.3,/)
C
C  FIND CONDITIONS IN REGION 4:
C
      EM1SQ = EM1*EM1
      TOM = 2./GM1/EM1SQ
      OPTOM = 1. + TOM
      U4BU1 = 1.
      KOUNT = 1
    5 CONTINUE
      OLD = U4BU1
      ARG = (TAU*OPTOM - U4BU1*U4BU1)/TOM
      IF(ARG.GT.0.) GO TO 22
      PRINT*,'NEGATIVE ARGUMENT IN U4/U1 ITERATIONS'
      GO TO 1
   22 U4BU1 = ARG**(-GM1INV)
      ERR = ABS(U4BU1-OLD)
      IF(ERR.LT.1.E-07) GO TO 6
      KOUNT = KOUNT + 1
      IF(KOUNT.LE.30) GO TO 5
      PRINT*,'U4/U1 ITERATIONS FAILED TO CONVERGE'
      GO TO 1
    6 R4BR1 = 1./U4BU1
      P4BP1 = R4BR1**GM
      T4BT1 = P4BP1/R4BR1
      A4BA1 = SQRT(T4BT1)
      T1BT01 = 1./(1. + GM1B2*EM1SQ)
      P1BP01 = T1BT01**GBGM1
      EM4 = U4BU1*EM1/A4BA1
      U4BA1 = EM4*A4BA1
```

```
      P4BP04 = (1. + GM1B2*EM4*EM4)**-GBGM1
      PRINT*,'U4/A1  R4/R1  P4/P1  T4/T1  A4/A1  M4  P04/P01'
      PRINT*,U4BA1,R4BR1,P4BP1,T4BT1,A4BA1,EM4,PR
       IREG = 4
      WRITE(2,101) IREG,U4BA1,R4BR1,P4BP1,T4BT1,A4BA1,EM4,PR
  101 FORMAT(1X,'REGION U4/A1  R4/R1  P4/P1  T4/T1  A4/A1  M4',
     * '  P04/P01',/I3,3X,7F7.4)
C
C  CHOOSE P2/P1, AND FIND THE CORRESPONDING CONDITIONS IN REGION 2:
C
       PRINT*,'INPUT P2/P1 (99. TO STOP)'
       READ*,P2BP1
       IF(P2BP1.EQ.99.) GO TO 30
       IF(P2BP1.GE.1.) GO TO 565
C  INCIDENT WAVE IS AN EXPANSION:
      US1BA1 = 1. + EM1
       R2BR1 = P2BP1**GMINV
       T2BT1 = P2BP1/R2BR1
       A2BA1 = SQRT(T2BT1)
       P02B01 = 1.
       U2BA1 = TBGM1*(A2BA1-1.) + EM1
       EM2 = U2BA1/A2BA1
       US2BA1 = A2BA1 + U2BA1
       GO TO 566
  565 US1BA1 = EM1 + SQRT((GP1*P2BP1+GM1)/2./GM)
       R2BR1 = (GP1*P2BP1 + GM1)/(GM1*P2BP1 + GP1)
       T2BT1 = P2BP1/R2BR1
       A2BA1 = SQRT(T2BT1)
       P02B01 = (P2BP1**(-GM1INV))*(R2BR1**GBGM1)
       EM2 = US1BA1/A2BA1 - SQRT((GM1*P2BP1+GP1)/(2.*GM*P2BP1))
       U2BA1 = EM2*A2BA1
  566 PRINT*,'US1/A1  U2/A1  R2/R1  P2/P1  T2/T1  A2/A1  M2  P02/P01'
      PRINT*,US1BA1,U2BA1,R2BR1,P2BP1,T2BT1,A2BA1,EM2,P02B01
       IREG = 2
       WRITE(2,102) IREG,US1BA1,U2BA1,R2BR1,P2BP1,T2BT1,A2BA1,EM2,
     *  P02B01
  102 FORMAT(/1X,'REGION US1/A1  U2/A1  R2/R1  P2/P1  T2/T1  A2/A1',
     * '  M2  P02/P01',/I3,3X,8F7.4)
      IF(US2BA1.NE.0.) WRITE(2,115) US2BA1
  115 FORMAT(3X,'US2/A1 =',F7.4)
C
C  NOW ITERATE ON P5/P4 AND P3/P2 UNTIL PRESSURES AND PARTICLE
C  VELOCITIES IN REGIONS 5 AND 6 ARE MATCHED
C
       ITER = 1
       KASE = 1
       P5BP4 = 1.
       P54A = 1.
       P54B = P2BP1
       P3BP2 = 1.
       P32A = 1.
       P32B = P2BP1
   10 CONTINUE
```

```
      IF(P2BP1.GT.1.) GO TO 301
C  FOR THE CASE OF AN INCIDENT EXPANSION:
      R5BR4 = P5BP4**GMINV
       T5BT4 = P5BP4/R5BR4
       A5BA4 = SQRT(T5BT4)
       US5BA4 = U4BA1 + A4BA1
       P05B04 = 1.
       U5BA4 = TBGM1*(A5BA4-1.) + EM4
       EM5 = U5BA4/A5BA4
       US5BA5 = US5BA4/A5BA4
      GO TO 302
C  FOR THE CASE OF AN INCIDENT SHOCK:
  301 US5BA4 = EM4 + SQRT((GP1*P5BP4 + GM1)/2./GM)
      R5BR4 = (GP1*P5BP4+GM1)/(GM1*P5BP4+GP1)
       T5BT4 = P5BP4/R5BR4
       A5BA4 = SQRT(T5BT4)
       P05B04 = (P5BP4**(-GM1INV))*(R5BR4**GBGM1)
       US5BA5 = US5BA4/A5BA4
       EM5 = US5BA5 - SQRT((GM1*P5BP4 + GP1)/2./GM/P5BP4)
  302 U5BA1 = EM5*A5BA4*A4BA1
       T5BT1 = T5BT4*T4BT1
       A5BA1 = SQRT(T5BT1)
       US5BA1 = US5BA5*A5BA1
       R5BR1 = R5BR4*R4BR1
       P5BP1 = P5BP4*P4BP1
       P05B01 = P05B04*PR
      PRINT*,'US5/A1  U5/A1  R5/R1  P5/P1  T5/T1  A5/A1  EM5  P05/P01'
      PRINT*,US5BA1,U5BA1,R5BR1,P5BP1,T5BT1,A5BA1,EM5,P05B01
      IF(KASE.EQ.2) GO TO 11
       X5A = P5BP1
       Y5A = U5BA1
       GO TO 63
   11 X5B = P5BP1
      Y5B = U5BA1
C
C  CHOOSE P3/P2, AND THE CORRESPOPNDING CONDITIONS IN REGIONS 3:
C
   63 IF(P3BP2.GT.0.) GO TO 563
      PRINT*,'P3/P2 NEGATIVE - PRESS RETURN TO CONTINUE'
       PAUSE
       WRITE(2,110)
  110 FORMAT(/5X,'P3/P2 NEGATIVE - GOING TO NEXT CASE',//)
       GO TO 1
  563 IF(P3BP2.LT.1.) GO TO 60
      US3BA2 = - EM2 + SQRT((GP1*P3BP2+GM1)/2./GM)
       R3BR2 = (GP1*P3BP2+GM1)/(GM1*P3BP2+GP1)
       T3BT2 = P3BP2/R3BR2
       A3BA2 = SQRT(T3BT2)
       P03B02 = (P3BP2**-GM1INV)*(R3BR2**GBGM1)
       US3BA3 = US3BA2/A3BA2
       EM3 = - US3BA3 + SQRT((GM1*P3BP2+GP1)/2./GM/P3BP2)
       T3BT1 = T3BT2*T2BT1
       A3BA1 = SQRT(T3BT1)
       US3BA1 = US3BA3*A3BA1
       R3BR1 = R3BR2*R2BR1
       P3BP1 = P3BP2*P2BP1
```

```
      P03B01 = P03B02*P02B01
      U3BA1 = EM3*A3BA1
      GO TO 66
   60 A3BA2 = P3BP2**GM1BTG
     A3BA1 = A3BA2*A2BA1
      T3BT1 = A3BA1*A3BA1
      U3BA1 = TBGM1*(A2BA1-A3BA1) + U2BA1
      P3BP1 = P3BP2*P2BP1
      US3BA1 = -A2BA1
      R3BR2 = P3BP2**GMINV
      R3BR1 = R3BR2*R2BR1
      EM3 = U3BA1/A3BA1
      P03B01 = P02B01
   66 CONTINUE
     PRINT*,'US3/A1  U3/A1  R3/R1  P3/P1  T3/TS   A3BA1  EM3  P03/P01'
     PRINT*,US3BA1,U3BA1,R3BR1,P3BP1,T3BT1,A3BA1,EM3,P03B01
C
C  NOW DO THE TRANSITION FROM REGION 3 TO REGION 6:
C
      P06B01 = PR*P03B01
      EM3SQ = EM3*EM3
      TOM = 2./GM1/EM3SQ
      OPTOM = 1. + TOM
      U6BU3 = 1.
      KOUNT = 1
   55 CONTINUE
     OLD = U6BU3
      ARG = ((TAU*OPTOM - U6BU3*U6BU3)/TOM)
      IF(ARG.GT.0.) GO TO 52
      PRINT*,'NEGATIVE ARGUMENT IN U6/U3 ITERATIONS'
      GO TO 1
   52 U6BU3 = ARG**(-GM1INV)
     ERR = ABS(U6BU3-OLD)
      IF(ERR.LT.1.E-07) GO TO 56
      KOUNT = KOUNT + 1
      IF(KOUNT.LE.30) GO TO 55
      PRINT*,'U6/U3 ITERATIONS FAILED TO CONVERGE'
      GO TO 1
   56 R6BR3 = 1./U6BU3
     P6BP3 = R6BR3**GM
      T6BT3 = P6BP3/R6BR3
      A6BA3 = SQRT(T6BT3)
      EM6 = U6BU3*EM3/A6BA3
      U6BA1 = U6BU3*U3BA1
      T6BT1 = T6BT3*T3BT1
      A6BA1 = SQRT(T6BT1)
      R6BR1 = R6BR3*R3BR1
      P6BP1 = P6BP3*P3BP1
     PRINT*,'U6/A1  R6/R1  P6/P1  T6/T1  A6/A1  M6  P06/P01'
     PRINT*,U6BA1,R6BR1,P6BP1,T6BT1,A6BA1,EM6,P06B01
     IF(KASE.EQ.2) GO TO 61
      X6A = P6BP1
      Y6A = U6BA1
      DX = X5A - X6A
      DY = Y5A - Y6A
```

```
      DIFA = DX*DX + DY*DY
      KASE = 2
      P5BP4 = P54B
      P3BP2 = P32B
      GO TO 10
   61 X6B = P6BP1
      Y6B = U6BA1
C
C  NOW FIND THE NEXT GUESS, AND ITERATE ONE MORE TIME:
C
      DX = X5B - X6B
      DY = Y5B - Y6B
      DIFB = DX*DX + DY*DY
      DIF = SQRT(DIFB)
      IF(DIF.LT.1.E-04) GO TO 70
      ITER = ITER + 1
      IF(ITER.LE.30) GO TO 564
      PRINT*,'ITERATIONS ON P3/P2 AND P5/P4 NOT CONVERGING - '
      PRINT*,'PRESS RETURN TO RESTART'
      PAUSE
      WRITE(2,111)
  111 FORMAT(2X,'ITERATIONS ON P3/P2 AND P5/P4 NOT CONVERGENT',/)
      GO TO 1
  564 SLP5 = (Y5B-Y5A)/(X5B-X5A)
      B5 = Y5A - SLP5*X5A
      SLP6 = (Y6B-Y6A)/(X6B-X6A)
      B6 = Y6A - SLP6*X6A
      XNEW = -(B5-B6)/(SLP5-SLP6)
      P54NEW = P54A + (P54B-P54A)*(XNEW-X5A)/(X5B-X5A)
      P32NEW = P32A + (P32B-P32A)*(XNEW-X6A)/(X6B-X6A)
      PRINT*,'ITER  P5/P4  P3/P2  DIF'
      PRINT*,ITER,P54NEW,P32NEW,DIF
      IF(U6BA1.LT.0.) PRINT*,'NOTE - U6 IS NEGATIVE'
      PAUSE
      IF(DIFA.LT.DIFB) GO TO 71
      X5A = X5B
      X6A = X6B
      Y5A = Y5B
      Y6A = Y6B
      P54A = P54B
      P32A = P32B
      DIFA = DIF*DIF
   71 P54B = P54NEW
      P32B = P32NEW
      P5BP4 = P54B
      P3BP2 = P32B
      GO TO 10
   70 CONTINUE
      WRITE(2,112) P5BP4
  112 FORMAT(/10X,'P5/P4 =',F7.4)
      IREG = 5
      WRITE(2,104) IREG,US5BA1,U5BA1,R5BR1,P5BP1,T5BT1,A5BA1,EM5,
     *   P05B01
```

```
104 FORMAT(1X,'REGION US5/A1  U5/A1  R5/R1  P5/P1  T5/T1  A5/A1',
   *  '  EM5  P05/P01',/I3,3X,8F7.4)
    IREG = 3
    WRITE(2,105) P3BP2,IREG,US3BA1,U3BA1,R3BR1,P3BP1,T3BT1,A3BA1,
   *   EM3,P03B01
105 FORMAT(/10X,'P3/P2 =',F7.4,
   *   /1X,'REGION US3/A1  U3/A1  R3/R1  P3/P1  T3/T1  A3/A1  EM3'
   *   '  P03/P01',/I3,3X,8F7.4)
    IREG = 6
    WRITE(2,106) IREG,U6BA1,R6BR1,P6BP1,T6BT1,A6BA1,EM6,P06B01
106 FORMAT(/1X,'REGION U6/A1  R6/R1  P6/P1  T6/T1  A6/A1  M6',
   *  '  P06/P01',/I3,3X,7F7.4)
     EMDOT1 = EM1
     EMDOT3 = U3BA1*R3BR1
     WRITE(2,108) EMDOT1,EMDOT3
108 FORMAT(10X,'MASSFLOW IN REGIONS 1 AND 4 =',F7.4,
   *   /10X,'MASS FLOW IN REGIONS 3,5, AND 6 =',F7.4,//)
    GO TO 1
 30 CONTINUE
    CLOSE(UNIT=2)
     STOP
     END
```

APPENDIX B

FORTRAN PROGRAM FOR UPSTREAM-PROPAGATING WAVES

```
C  PROGRAM WAVEC - CALCULATES THE TRANSMISSION AND REFLECTION OF
C  PRESSURE WAVES AT AN ACTUATOR DISK.
C  WAVEC HANDLES EXPANSIONS AND SHOCKS INCIDENT FROM THE DOWNSTREAM
C  SIDE OF THE ACTUATOR DISK
       OPEN(UNIT=2,FILE='WAVECOUT',STATUS='NEW')
     1 PRINT*,'INPUT STAGE PRESSURE RATIO (99. TO STOP),EM1,GAMMA'
      READ*,PR,EM1,GM
       IF(PR.EQ.99.) GO TO 30
       GMINV = 1./GM
       GM1 = GM - 1.
       GM1INV = 1./GM1
       TBGM1 = 2./GM1
       GBGM1 = GM/GM1
       GP1 = GM + 1.
       GM1BG = GM1/GM
       GM1B2 = GM1/2.
       GM1BTG = GM1BG/2.
       TAU = PR**GM1BG
       WRITE(2,100) PR,EM1,GM,TAU
   100 FORMAT(10X,'SOLUTION FOR PRESSURE WAVES INCIDENT FROM THE',
     *   ' DOWNSTREAM SIDE OF AN ACTUATOR',
     *   ' DISK',/10X,'OF STAGNATION-PRESSURE RATIO =',F5.1,/10X,
     *   'INLET MACH NUMBER =',F5.2,'  SPECIFIC-HEAT RATIO =',F5.2,
     *   /10X,'STAGNATION-TEMPERATURE RATIO IS',F7.3,/)
C
C  FIND CONDITIONS IN REGION 4:
C
       EM1SQ = EM1*EM1
       TOM = 2./GM1/EM1SQ
       OPTOM = 1. + TOM
       U4BU1 = 1.
       KOUNT = 1
     5 CONTINUE
       OLD = U4BU1
       ARG = (TAU*OPTOM - U4BU1*U4BU1)/TOM
       IF(ARG.GT.0.) GO TO 22
       PRINT*,'NEGATIVE ARGUMENT IN U4/U1 ITERATIONS'
       GO TO 1
    22 U4BU1 = ARG**(-GM1INV)
      ERR = ABS(U4BU1-OLD)
       IF(ERR.LT.1.E-07) GO TO 6
       KOUNT = KOUNT + 1
       IF(KOUNT.LE.30) GO TO 5
       PRINT*,'U4/U1 ITERATIONS FAILED TO CONVERGE'
       GO TO 1
     6 R4BR1 = 1./U4BU1
      P4BP1 = R4BR1**GM
      T4BT1 = P4BP1/R4BR1
      A4BA1 = SQRT(T4BT1)
```

```
      T1BT01 = 1./(1. + GM1B2*EM1SQ)
      P1BP01 = T1BT01**GBGM1
      EM4 = U4BU1*EM1/A4BA1
      U4BA1 = EM4*A4BA1
      P4BP04 = (1. + GM1B2*EM4*EM4)**-GBGM1
      PRINT*,'U4/A1  R4/R1  P4/P1  T4/T1  A4/A1  M4  P04/P01'
      PRINT*,U4BA1,R4BR1,P4BP1,T4BT1,A4BA1,EM4,PR
       IREG = 4
      WRITE(2,101) IREG,U4BA1,R4BR1,P4BP1,T4BT1,A4BA1,EM4,PR
  101 FORMAT(1X,'REGION U4/A1  R4/R1  P4/P1  T4/T1  A4/A1  M4',
     * '  P04/P01',/I3,3X,7F7.4)
C
C  CHOOSE P5/P4, AND FIND THE CORRESPONDING CONDITIONS IN REGION 5:
C
       PRINT*,'INPUT P5/P4 (99. TO STOP)'
       READ*,P5BP4
       IF(P5BP4.EQ.99.) GO TO 30
       IF(P5BP4.GE.1.) GO TO 565
C  INCIDENT WAVE IS AN EXPANSION:
      US5BA4 = EM4 - 1.
       P5BP1 = P5BP4*P4BP1
      R5BR4 = P5BP4**GMINV
       R5BR1 = R5BR4*R4BR1
      T5BT4 = P5BP4/R5BR4
       T5BT1 = T5BT4*T4BT1
      A5BA4 = SQRT(T5BT4)
       A5BA1 = A5BA4*A4BA1
      P05B04 = 1.
       P05B01 = PR
      U5BA4 = TBGM1*(1.-A5BA4) + EM4
       U5BA1 = U5BA4*A4BA1
      EM5 = U5BA4/A5BA4
       GO TO 566
C  INCIDENT WAVE IS A SHOCK:
  565 US5BA4 = EM4 - SQRT((GP1*P5BP4+GM1)/2./GM)
       R5BR4 = (GP1*P5BP4 + GM1)/(GM1*P5BP4 + GP1)
       R5BR1 = R5BR4*R4BR1
       P5BP1 = P5BP4*P4BP1
       T5BT4 = P5BP4/R5BR4
       T5BT1 = T5BT4*T4BT1
       A5BA4 = SQRT(T5BT4)
       A5BA1 = A5BA4*A4BA1
       P05B04 = (P5BP4**(-GM1INV))*(R5BR4**GBGM1)
       P05B01 = P05B04*PR
       US5BA5 = US5BA4/A5BA4
       EM5 = US5BA5 + SQRT((GM1*P5BP4+GP1)/(2.*GM*P5BP4))
       U5BA1 = EM5*A5BA1
  566 PRINT*,'US5/A4  U5/A1  R5/R1  P5/P1  T5/T1  A5/A1  M5  P05/P01'
      PRINT*,US5BA4,U5BA1,R5BR1,P5BP1,T5BT1,A5BA1,EM5,P05B01
       IREG = 5
       WRITE(2,102) IREG,US5BA4,U5BA1,R5BR1,P5BP1,T5BT1,A5BA1,EM5,
     *  P05B01
  102 FORMAT(/1X,'REGION US5/A4  U5/A1  R5/R1  P5/P1  T5/T1  A5/A1',
     * '  M5  P05/P01',/I3,3X,8F7.4)
```

```
C
C  NOW ITERATE ON P2/P1 AND P6/P5 UNTIL PRESSURES AND PARTICLE
C  VELOCITIES IN REGIONS 6 AND 7 ARE MATCHED
C
      ITER = 1
      KASE = 1
      P2BP1 = 1.
      P21A = 1.
      P21B = P5BP4
      P6BP5 = 1.
      P65A = 1.
      P65B = P5BP4
   10 CONTINUE
      IF(P5BP4.GT.1.) GO TO 301
C  FOR THE CASE OF AN INCIDENT EXPANSION:
      R2BR1 = P2BP1**GMINV
      T2BT1 = P2BP1/R2BR1
      A2BA1 = SQRT(T2BT1)
      US2BA1 = EM1 - 1.
      P02B01 = 1.
      U2BA1 = TBGM1*(1.-A2BA1) + EM1
      EM2 = U2BA1/A2BA1
      US2BA2 = US2BA1/A2BA1
      GO TO 302
C  FOR THE CASE OF AN INCIDENT SHOCK:
  301 US2BA1 = EM1 - SQRT((GP1*P2BP1 + GM1)/2./GM)
      R2BR1 = (GP1*P2BP1+GM1)/(GM1*P2BP1+GP1)
      T2BT1 = P2BP1/R2BR1
      A2BA1 = SQRT(T2BT1)
      P02B01 = (P2BP1**(-GM1INV))*(R2BR1**GBGM1)
      US2BA2 = US2BA1/A2BA1
      EM2 = US2BA2 + SQRT((GM1*P2BP1 + GP1)/2./GM/P2BP1)
  302 U2BA1 = EM2*A2BA1
      PRINT*,'US2/A1  U2/A1  R2/R1  P2/P1  T2/T1  A2/A1  EM2  P02/P01'
      PRINT*,US2BA1,U2BA1,R2BR1,P2BP1,T2BT1,A2BA1,EM2,P02B01

C  NOW DO THE TRANSITION FROM REGION 2 TO REGION 7:
C
      P07B01 = PR*P02B01
      EM2SQ = EM2*EM2
      TOM = 2./GM1/EM2SQ
      OPTOM = 1. + TOM
      U7BU2 = 1.
      KOUNT = 1
   55 CONTINUE
      OLD = U7BU2
      ARG = ((TAU*OPTOM - U7BU2*U7BU2)/TOM)
      IF(ARG.GT.0.) GO TO 52
      PRINT*,'NEGATIVE ARGUMENT IN U7/U2 ITERATIONS'
      GO TO 1
   52 U7BU2 = ARG**(-GM1INV
      ERR = ABS(U7BU2-OLD)
      IF(ERR.LT.1.E-07) GO TO 56
      KOUNT = KOUNT + 1
      IF(KOUNT.LE.30) GO TO 55
      PRINT*,'U7/U2 ITERATIONS FAILED TO CONVERGE'
      GO TO 1
```

```
   56 R7BR2 = 1./U7BU2
      P7BP2 = R7BR2**GM
       T7BT2 = P7BP2/R7BR2
       A7BA2 = SQRT(T7BT2)
       EM7 = U7BU2*EM2/A7BA2
       U7BA1 = U7BU2*U2BA1
       T7BT1 = T7BT2*T2BT1
       A7BA1 = SQRT(T7BT1)
       R7BR1 = R7BR2*R2BR1
       P7BP1 = P7BP2*P2BP1
      PRINT*,'U7/A1  R7/R1  P7/P1  T7/T1  A7/A1  M7  P07/P01'
      PRINT*,U7BA1,R7BR1,P7BP1,T7BT1,A7BA1.EM7,P07B01
      IF(KASE.EQ.2) GO TO 11
       X7A = P7BP1
       Y7A = U7BA1
       X2A = P2BP1
       Y2A = U2BA1
       GO TO 63
   11 X7B = P7BP1
      Y7B = U7BA1
       X2B = P2BP1
       Y2B = U2BA1
C
C  CHOOSE P6/P5
C
   63 IF(P6BP5.GT.0.) GO TO 563
      PRINT*,'P6/P5 NEGATIVE - PRESS RETURN TO CONTINUE'
       PAUSE
       WRITE(2.110)
  110 FORMAT(/5X,'P6/P5 NEGATIVE - GOING TO NEXT CASE',//)
       GO TO 1
  563 IF(P6BP5.LT.1.) GO TO 60
      US6BA5 =  EM5 + SQRT((GP1*P6BP5+GM1)/2./GM)
       R6BR5 = (GP1*P6BP5+GM1)/(GM1*P6BP5+GP1)
       T6BT5 = P6BP5/R6BR5
       A6BA5 = SQRT(T6BT5)
       P06B05 = (P6BP5**-GM1INV)*(R6BR5**GBGM1)
       US6BA6 = US6BA5/A6BA5
       EM6 =  US6BA6 - SQRT((GM1*P6BP5+GP1)/2./GM/P6BP5)
       T6BT1 = T6BT5*T5BT1
       A6BA1 = SQRT(T6BT1)
       US6BA1 = US6BA6*A6BA1
       R6BR1 = R6BR5*R5BR1
       P6BP1 = P6BP5*P5BP1
       P06B01 = P06B05*P05B01
       U6BA1 = EM6*A6BA1
       GO TO 66
   60 A6BA5 = P6BP5**GM1BTG
      A6BA1 = A6BA5*A5BA1
       T6BT1 = A6BA1*A6BA1
       U6BA1 = TBGM1*(A6BA1-A5BA1) + U5BA1
       P6BP1 = P6BP5*P5BP1
       US6BA1 = -A5BA1
       R6BR5 = P6BP5**GM1INV
       R6BR1 = R6BR5*R5BR1
```

```
      EM6 = U6BA1/A6BA1
      P06B01 = P05B01
   66 CONTINUE
      PRINT*,'US6/A1  U6/A1  R6/R1  P6/P1  T6/T1   A6BA1  EM6  P06/P01'
       PRINT*,US6BA1,U6BA1,R6BR1,P6BP1,T6BT1,A6BA1,EM6,P06B01
C
      IF(KASE.EQ.2) GO TO 61
       X6A = P6BP1
       Y6A = U6BA1
       DX = X6A - X7A
       DY = Y6A - Y7A
       DIFA = DX*DX + DY*DY
       KASE = 2
       P2BP1 = P21B
       P6BP5 = P65B
       GO TO 10
   61 X6B = P6BP1
      Y6B = U6BA1
C
C  NOW FIND THE NEXT GUESS, AND ITERATE ONE MORE TIME:
C
       DX = X6B - X7B
       DY = Y6B - Y7B
       DIFB = DX*DX + DY*DY
       DIF = SQRT(DIFB)
       IF(DIF.LT.1.E-04) GO TO 70
       ITER = ITER + 1
       IF(ITER.LE.30) G' 70 564
       PRINT*,'ITERATIONS ON P6/P5 AND P2/P1 NOT CONVERGING - '
       PRINT*,'PRESS RETURN TO RESTART'
       PAUSE
       WRITE(2,111)
  111 FORMAT(2X,'ITERATIONS ON P6/P5 AND P2/P1 NOT CONVERGENT',/)
       GO TO 1
  564 SLP6 = (Y6B-Y6A)/(X6B-X6A)
       B6 = Y6A - SLP6*X6A
       SLP7 = (Y7B-Y7A)/(X7B-X7A)
       B7 = Y7A - SLP7*X7A
       XNEW = -(B6-B7)/(SLP6-SLP7)
       P71NEW = XNEW
       P21NEW = P21A + (XNEW-X7A)*(P21B-P21A)/(X7B-X7A)
       P65NEW = XNEW/P5BP1
      PRINT*,'ITER  P2/P1  P6/P5  DIF'
      PRINT*,ITER,P21NEW,P65NEW,DIF
       IF(U6BA1.LT.0.) PRINT*,'NOTE - U6 IS NEGATIVE'
       PAUSE
      IF(DIFA.LT.DIFB) GO TO 71
       X6A = X6B
       X7A = X7B
       Y6A = Y6B
       Y7A = Y7B
       P21A = P21B
       P65A = P65B
       DIFA = DIF*DIF
```

```
 71 P21B = P21NEW
    P65B = P65NEW
     P2BP1 = P21B
     P6BP5 = P65B
     GO TO 10
 70 CONTINUE
    WRITE(2,112) P2BP1
112 FORMAT(/10X,'P2/P1 =',F7.4)
    IREG = 2
    WRITE(2,104) IREG,US2BA1,U2BA1,R2BR1,P2BP1,T2BT1,A2BA1,EM2,
   *   P02B01
104 FORMAT(1X,'REGION US2/A1  U2/A1  R2/R1  P2/P1  T2/T1  A2/A1',
   *  '  EM2  P02/P01',/I3,3X,8F7.4)
    IREG = 6
    WRITE(2,105) P6BP5,IREG,US6BA1,U6BA1,R6BR1,P6BP1,T6BT1,A6BA1,
   *   EM6,P06B01
105 FORMAT(/10X,'P6/P5 =',F7.4,
   *   /1X,'REGION US6/A1  U6/A1  R6/R1  P6/P1  T6/T1  A6/A1  EM6'
   *   '  P06/P01',/I3,3X,8F7.4)
    IREG = 6
    WRITE(2,106) IREG,U6BA1,R6BR1,P6BP1,T6BT1,A6BA1,EM6,P06B01
106 FORMAT(/1X,'REGION U6/A1  R6/R1  P6/P1  T6/T1  A6/A1  M6',
   *  '  P06/P01',/I3,3X,7F7.4)
     EMDOT1 = EM1
     EMDOT2 = U2BA1*R2BR1
     WRITE(2,108) EMDOT1,EMDOT2
108 FORMAT(10X,'MASSFLOW IN REGIONS 1 AND 4 =',F7.4,
   *   /10X,'MASS FLOW IN REGIONS 2,7, AND 6 =',F7.4,//)
    GO TO 1
 30 CONTINUE
    CLOSE(UNIT=2)
     STOP
     END
```

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